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Name: Chester H. Bullock  
Louis M. Collier

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Institution: Oklahoma State University

Stillwater, Oklahoma

Title of Study: A MODERN PROGRAM IN MATHEMATICS FOR THE SECONDARY SCHOOLS


Number of pages in Study: 31 Candidates for Degree of Master of Science

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Nature of Study: There exists today a tremendous amount of mathematical information. During the past two or three decades, the pace of acceleration has greatly increased because of a much greater demand for mathematicians, engineers and technicians. The acceleration has passed beyond the ability of the secondary school to dispense the information. Traditional mathematics taught by curriculum arrangements of the past has been found lacking. The public in general and educators in particular are now definitely in favor of a change to modern mathematics as needed in 20th century.

Use of the Study: The information contained in this report could be used as a lecture to interested groups or to point out to administrators the great need of change toward modern mathematical concepts, curricula and teacher-training programs.

ADVISER'S APPROVAL

  
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A MODERN PROGRAM IN MATHEMATICS  
FOR THE SECONDARY SCHOOLS

By

Chester H. Bullock  
Bachelor of Science  
Southern State College  
Magnolia, Arkansas  
1952

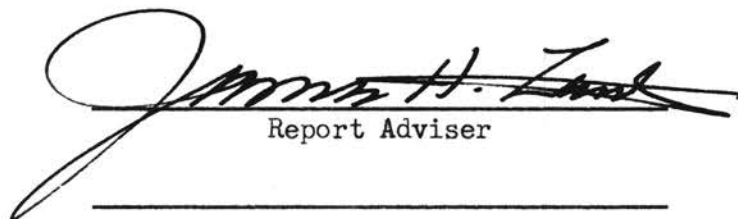
and

Louis M. Collier  
Bachelor of Science  
Grambling College  
Grambling, Louisiana  
1954

Submitted to the Faculty of the Graduate School of the  
Oklahoma State University  
in partial fulfillment of the requirements  
for the degree of  
MASTER OF SCIENCE  
May, 1960

A MODERN PROGRAM IN MATHEMATICS  
FOR THE SECONDARY SCHOOLS

Report Approved:

  
Report Adviser

  
Dean of the Graduate School

## ACKNOWLEDGEMENTS

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## CHAPTER I

### INTRODUCTION

The purpose of this report is to propose a modern program in mathematics for the secondary school. The report is prepared with several aims in mind. Probably the first aim of the report was to aid the writers themselves. A second aim was to gather facts and figures which might aid others seeking the same type of information. A third aim lies in the hope that secondary schools may consider this type of information and proposals as they candidly examine their curricula. A fourth aim may be that the bibliography of such a report could serve as a guide for those needing further reading materials. The writers themselves have learned of many books and publications of which they were ignorant heretofore. Information about possible materials and supplies have been brought to mind. If the report could be used to benefit school, teacher or pupil (or all three), the writers would feel more than amply repaid for any trouble encountered or energy expended in the preparation of it.

As we consider the various aspects of modern mathematics, we hear the question--what's new? These words could well serve as the appropriate greeting of mathematicians everywhere. Mathematicians, however, are not the only ones who are concerned, vitally concerned, today with this question which is so fraught with unlimited meaning and implication.

The words to the effect that mathematics is entering the greatest era this venerable discipline has ever known seem to express the feeling

(as well as the sincere desire) of all who are familiar with the past and also the future of mathematics as a whole.<sup>1</sup> The words of another who is actively engaged in this battle of change say that

though mathematics has been developing for 5000 years, the subject has never been as lively as it is today. The pace of mathematical discovery and invention has accelerated amazingly during the last 20 years. It has been said that mathematics is the only branch of learning in which all of the theories of 2000 years ago are still valid. Hence, there is little that is basically wrong with the mathematics of the past, and the vitality and vigor of present day mathematical research indicates that the sheer bulk of mathematical developments is staggering.<sup>2</sup>

Modern mathematics has come to stay. The teacher of mathematics must of necessity prepare himself for the transition from traditional mathematics of the past to the modern contemporary mathematics of the future, but keeping in mind the words already quoted from Dr. Zant. Perhaps the changes that came about in the past were more or less chance occurrences, but changes today arise to a great extent through demand. Some have thought of these changes occurring today as a part of the plan of survival and perhaps rightly so. No one can put a true value on mathematics.

In no sense of the word could one say today that mathematics is static; neither can it be truthfully stated that mathematics is only a discipline, that is, mathematics for the sake of mathematics. Mathematics is everywhere; it is so common that some are not even conscious of their

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<sup>1</sup>S. S. Wilks, "New Fields and Organization in the Mathematical Sciences," The Mathematics Teacher, January, 1958, LI, p. 2.

<sup>2</sup>James H. Zant, "New World of Mathematics," Oklahoma State Alumnus, February, 1960, p. 8.

use of it, and yet, one five-cent pencil cannot be purchased without using many mathematical ideas. In the search for what's new in mathematics,

the foundations have been, and continue to be, subjected to critical examination. Some traditional techniques have been discarded as no longer efficient, others have undergone radical modification. Familiar definitions of many basic concepts have been dropped as passe and inexact, to be replaced by qualifications more modern in import. New structures have been postulated and subjected to critical examination, some to be discarded, some to be modified, some to be retained.<sup>3</sup>

It has been said that since October 4, 1957, mathematicians have been in the center of the spotlight.<sup>4</sup> Truly, they are among those who live in the proverbial glass houses. Everyone is watching; the mathematician himself needs to take stock of the situation. So, what are the characteristics of a mathematician? He is one who knows that there is no culture sans mathematics even as there is no explanation sans mathematics. To the one who sees otherwise, let him try to explain something, anything, without the use of mathematical ideas. A mathematician is one who is not afraid of mental challenges and the expending of much mental energy in meeting those challenges. He is one who sees elegance in mathematical works. The elegance of a theorem is, according to Dr. Polya of Stanford University, directly proportional to the number of ideas you can see in it and inversely proportional to the effort it takes to see them.<sup>5</sup>

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<sup>3</sup>Lynwood F. Wren, "What's New in Mathematics," The Mathematics Teacher, November, 1956, XLIX, pp. 555-556.

<sup>4</sup>Ray C. Maul, "Let's Look at the New Mathematics and Science Teachers," The Mathematics Teacher, November, 1958, LI, pp. 531-534.

<sup>5</sup>George A. W. Boehm, The New World of Mathematics, (New York, 1959), p. 21.



Again, a mathematician is one who will sincerely examine and question, discard, and conclude all on logical reasoning, for he is unafraid of the opinions of others once he has found mathematical truth.

Mathematics is surely on the move. This, then, is not the big question. The big question, rather, is who is going to move along with mathematics. Mathematics and mathematicians are very indispensable today on every hand in every activity. Progress itself would soon cease if there were no more mathematical minds developed and trained.

## CHAPTER II

### AREAS OF INVESTIGATION

The areas investigated during the preparation of this report were very numerous, so numerous in fact that only a few of the more prominent ones will bear mentioning due to the space involved. Perhaps the first consideration should rightly be the teacher, for mathematics, nor any other subject, would not long enjoy its place in the curriculum without good teaching. Proof that the eyes of all are at present focused upon the mathematics and science teachers all over the world is evidenced by the many articles appearing in publications as well as the many committee reports that are being made. The programs of study around the world are being compared with those in the United States. The need was never greater for an honest, searching evaluation. Even more than others, the mathematics teacher must, because of many factors involved in his task, serve as coordinator, guide, motivator, counselor, dispenser of good supplementary materials, as well as teacher. In the opinion of the writers, the mathematics teacher's task lies almost wholly apart from the work of his fellow teachers because of their inability to help the students cope with mathematical ideas.

Are those in the teaching profession today prepared to teach modern mathematics? Perhaps not, but the many opportunities afforded today's teachers will soon make it inexcusable for one to plead ignorance of these modern trends and concepts. With a reasonable amount of preparation and

readings from among the many journals discussing these modern concepts, teachers of mathematics may lead on to the final goal. There are many questions about mathematics which can no longer be ignored. When we can satisfy ourselves on such questions, then we can say that our program is what it should be; but, as long as some of these questions remain unanswered, then there is a vital need of modification.

The teacher of mathematics must be a person of vast knowledge and deep insight. He must be able to interpret data and problems. He must be able to refer quickly and accurately to materials in the library, supplementary materials, as well as the vast writings in journals about mathematics and many other subjects. Many times the mathematics teacher is woefully handicapped in that he is not provided with even the barest of needed materials and equipment for his room. That day is surely fast passing when the teacher was given a text, a box of crayon, a meter stick, and a piece of string with which to draw circles and turned loose to face a class of perhaps 35 eager young people. The one who, even under such adverse teaching arrangements as these, can still do a commendable job of teaching and motivation deserves some kind of medal of honor for he has served over and above the call of duty.

With limited materials and equipment, it is no small wonder that students leave the course still unsatisfied in their quest for knowledge. The program, whether traditional or modern, must be geared to and revolve around the student and his need. Many of the better qualified students, upon whom the future to a great extent is said to rest, have been left unchallenged and unstimulated by such programs in the past.

The preparation of future teachers trained to handle the new program in mathematics should be relatively simple since new courses can be designed and offered in their college training. In fact, such courses are already being

given in some colleges. The problem of the teacher already in service, however, is more difficult, since he or she must acquire a knowledge of the new mathematics while doing a daily teaching job.

The answer here lies in the many and varied programs for teachers. These programs can and should be adapted to give our present mathematics teachers a working knowledge of the materials needed to put the Commission's program into use. Summer courses, special conferences and institutes, study groups, professional meetings, self-study, teacher's guides, and<sup>1</sup> new books, all offer ways to meet and solve this problem.

Thus, it is seen that a teacher who is ambitious enough to take advantage of these programs and ideas may prepare himself to more effectively teach his subject even in the more complex and technological world of today.

The world of today demands more mathematical knowledge on the part of more people than the world of yesterday and the world of tomorrow will make still greater demands. Our society leans more and more heavily on science and technology. The number of our citizens skilled in mathematics must be greatly increased; an understanding of the role of mathematics in our society is now a prerequisite for intelligent citizenship. Since no one can predict with certainty his future profession, much less foretell which mathematical skills will be required in the future by a given profession, it is important that mathematics be so taught that students will be able in later life to learn the new mathematical skills which the future will surely demand of many of them.

To achieve this objective in the teaching of school mathematics three things are required. First, we need an improved curriculum which will offer students not only the basic mathematical skills but also a deeper understanding of the basic concepts and structures of mathematics. Second, mathematics programs must attract and train more of those students who are capable of studying mathematics with profit. Finally, all help possible must be provided for teachers who are preparing themselves to teach these challenging and interesting courses.<sup>2</sup>

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<sup>1</sup>Commission On Mathematics, "A Summary of the Report of the Commission on Mathematics," 1959, p. 10.

<sup>2</sup>School Mathematics Study Group, Newsletter, March 1959, p. 4.

Many curricula throughout the country are outmoded and outdated. It is a sad fact that many, at least seemingly, are not interested in doing anything about them. The small school has been forced to alternate courses from year to year because of too few students or the shortage of teachers. The school that has been limited perhaps because of the lack of finances to equip and furnish proper facilities for departments can be found in almost any part of the country. Wide range of choice was not possible where teachers were untrained. Vital parts of the courses were skipped or hastily scanned and too much time was spent on insignificant parts of the text materials. In many schools, pupils were, early in their high-school careers, separated into two course areas sometimes called the Y-plan. If a pupil's interests can change, can he change his charted course without having had the intervening courses which may have been prerequisites?

As far as grouping is concerned, perhaps the best way, at least a good way, would be to segregate students in accordance with their future interests. Even at the high school level, it can be ascertained with some degree of accuracy what the capabilities and interests of most pupils are. This type of arrangement would eliminate the practice of giving everyone the same quality of materials and techniques, which if continued could easily bore some to distraction while leaving many others unstimulated and uninterested.

Surely the aims of mathematics need to be considered. Many of our high school students are not going into college and yet much of the high school mathematics program is directed to the end of preparing students for college. Much time is spent in emphasizing and drilling in techniques which, if not meaningless, are not applicable in modern living.

The subject matter, most of it developed over 300 years ago (in the case of plane and solid geometry it was over 2000 years ago), was chosen and its presentation organized in accordance with an attitude toward mathematics which no longer holds, and which has been discarded by present day mathematicians. With this curriculum and point of view, mathematics is presented mainly as a collection of slightly related techniques and manipulations. The profound, yet simple concepts get little attention. It has been said that "if art appreciation were taught in the same way, it would consist mostly of learning how to chip stone and mix paint."<sup>3</sup>

In spite of all that we may do, there will be some overlapping in the curricula of high school and colleges. Separation of the two is a necessary thing even if the line of distinction is not clearly defined. Perhaps when it is clearly seen and appreciated in its fullness that mathematics is indeed a way of thinking and reasoning, a structure or pattern, mathematics will get its place in the sun.

To teach mathematics in terms of structure and pattern rather than in terms of rules and tricks, will be greatly facilitated by a curriculum organized from that point of view. Those of us who have tried this sort of teaching realize that such an organization is probably essential. Hence, if we are to meet the needs of the present day world and students who live in it, we need a curriculum revised in content, but even more basically revised in point of view.<sup>4</sup>

The curricula of the past, even of the present, have not been revised at a rate that would keep fully abreast of the tremendous amount of mathematical knowledge. In fact, today the store of information is far beyond and ahead of the means of dispensing it at the level of the secondary school. Those who know the value and import of this knowledge

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<sup>3</sup>James H. Zant, "New World of Mathematics," Oklahoma State Alumnus, February, 1960, p. 10.

<sup>4</sup>James H. Zant, "Impact of Modern Mathematics on the Secondary School Curriculum," p. 4, Chapter V.

to the world as a means of progress and advancement are contending, therefore, for an immediate and effective revision of secondary school curricula. This fact arises because it is known that there is an overwhelming shortage of mathematicians, scientists and engineers, upon whom the resolving of many of the 20th century problems rests. These specialists in their respective fields admit the fact that scientific prowess alone, even though vital to the well-being and preservation of our way of life, is not enough. The future depends upon the ability of peoples to work together as families, as small communities, as states and as nations. So a change requiring more detailed study is in order.

As mathematics is evaluated, instruction of it must be considered, for mathematics is the collection of all facts resulting from logically flawless thinking.<sup>5</sup> If mathematics is in the spotlight, the instruction of it is also there. The real role of mathematics has probably never been fully conceived. The statement that teachers already in the profession and those being graduated at the present time are not fully prepared to present modern mathematical concepts would probably go unchallenged. There is debate as to the level at which low quality teaching of mathematics exists, and yet, all seem to agree that it does exist at some point, or points, along the way from elementary school to college level. The National Council of Teachers of Mathematics Curriculum Committees, as well as others, have studied ways and means of coordinating the entire mathematical structure and the facilitation of the instruction at all levels. Congress is much concerned about the instruction of

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<sup>5</sup>S. T. Sanders, "The Evaluation of Mathematics," National Mathematics Magazine, April, 1945, XIX, p.340.

mathematics and science and the National Science Foundation is greatly expanding its program of institutes.

The consensus of opinion relative to texts seemed to be in favor of readability, elimination of outmoded data, greater emphasis of structure, the inclusion of more modern concepts, and the proper correlation between all points included.



## CHAPTER III

### TRADITIONAL MATHEMATICS

The term traditional mathematics as being used here designates the mathematics that has developed over the past 5000 years as opposed to the more modern concepts which have developed mainly during the last few decades. This latter type of mathematics is modern mathematics, which would have had an earlier beginning if the works of some had not been passed over lightly. Unfortunately, this has been the case in many other fields as well as in mathematics.

A previous quotation by Dr. Zant that there is little basically wrong with the mathematics of the past gives a hint as to the content of traditional mathematics. Concepts came about probably more by accident than any other way. There was no hurry or demand for newer ideas, in fact, what was already known seemed to suffice for any and all occasions. Such views and attitudes, if allowed to govern school curricula would permit teaching mathematics as unrelated concepts. Mathematical structure would get little or no attention. Little attention would be given to fundamental concepts. Rigorous proofs could be omitted very easily. Even today, many, perhaps, skip certain fundamental sections because "it is hard to get pupils to see this," or "it is not needed by many of the pupils," or "I don't know too much about that myself."

The advanced thinking of 20th century America demands modern mathematics geared to the needs of today; stone-age mathematics won't do. The

basic ideas of the past are still true and worthy of thought but the methods of presenting materials, the point of view and the content need to be modernized.

Since there is a far greater demand for trained technicians today than ever before, their training should qualify them for their role in whatever field they plan to enter. This demand for scientists, mathematicians and engineers places a demand back upon the schools to better prepare them for their work by placing within their reach, as far as mathematics is concerned, 20th century courses taught and emphasized as such.

Limitations (deficiencies, if you wish to call them such) have arisen for many reasons, some of which, in all fairness, are beyond the control of the schools. Modern mathematics is relatively new; traditional mathematics is old. At least we like to think it is hard to change quickly from the old to the new. The change requires new texts, which are just now being made available. Library materials must be made available to correlate with the new approach to modern mathematics. Perhaps other new equipment and space is also necessary. All of these elements of change require tax dollars; if spent for these items there may be shortages in other departments.

As viewed by an English schoolmaster, mathematics should be taught as a whole, integrating the different branches into a comprehensive subject.<sup>1</sup> So, if some deficiencies do exist, there is a decided need for a turn toward modern mathematics as corrections are being made. The tra-

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<sup>1</sup>W. S. Brace, "An English Schoolmaster Looks at American Mathematics Teaching," The Mathematical Gazette, May, 1955, XXXIX, p. 92.

ditional treatment of the subject has not produced the desired results.

Several specific deficiencies in the current high school program are pointed out by Dr. E. J. Cogan.<sup>2</sup> The imagination of students is not captured because they do not know which techniques are important or why they are important. Teachers are inefficient at motivating the student in his studies and ineffective at satisfying those students whose questions show more than the usual depth and interest. The power of mathematics in fields different from sciences is not sufficiently emphasized.

These deficiencies pointed out seem to revolve around the teacher to a great extent. Regardless, however, of the cause of these deficiencies, the teachers themselves must take the lead in reorganization of the curriculum for they know best how to retain the good of the past to correlate with the newer concepts of the contemporary program to be instituted.

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<sup>2</sup>E. J. Cogan, "A New Approach to High School Mathematics," The Mathematics Teacher, May, 1957, L, p. 347.

## CHAPTER IV

### SUGGESTED PROGRAM

The purposes of such a modern program in mathematics are to contribute to individual development and to improve mathematical instruction by emphasizing:

1. Understanding basic structure, principles, historical developments, use and applications of mathematics.
2. Facility in the use of the fundamental processes of mathematical computations.
3. Growth in effective thinking and deductive reasoning.
4. Changes in the traditional mathematical program that are not too **radical** but will include the most recent concepts and will lead to a **clarification** of other concepts studied by the student.
5. Presentation of the mathematical content from a more modern point of view and concept that will equip the college capable student with such knowledge and skills as will be needed by the student in future study and will make the less talented student an effective and worthy community member.
6. Mathematics as a fundamental need of youths in the development of responsible citizenship and more effective living in an ever increasing technological society.

The needs of pupils in a modern school system develop and expand with the increasing complexity of the world in which we live, therefore,

it is important that there be periodic re-appraisals of the requirements, course offerings, course content, and the overall point of view in order that the best possible use be made of instructional time and that maximum results are attained.<sup>1</sup>

The content and sequence of offerings given in this outline are based on a study of the modern programs in mathematics which are now in operation and which were recommended particularly by the Commission on Mathematics and the School Mathematics Study Group. The outline includes the basic proposals of both groups so arranged as to involve as little overlapping as possible and yet contain all the topics suggested in the original proposals.

This program concerns itself mainly with the mathematics of grades 9 through 12, but since the secondary school program is necessarily predicated on certain learnings expected of students prior to entering the secondary school program, certain subject matter is regarded as essential to give the 7th and 8th grade students a proper foundation for the modern mathematics program.

The Commission on Mathematics made no specific proposals for grades 7 and 8 but did include in the report a brief statement on prerequisite mathematics.

The School Mathematics Study Group worked with the University of Maryland project and has proposed the following experimental units, the titles of which are listed below, but they are not separated by grades.

Mathematics for grades 7 and 8:

- I. An introductory unit on "why study mathematics."

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<sup>1</sup>Instructional Guide for High School Mathematics, Los Angeles City School District, Los Angeles, Calif., 1956, p.3.

- II. Decimals and non-decimal numerations.
- III. The natural numbers and zero.
- IV. Factoring and primes.
- V. Unsigned rationals.
- VI. Non-metric geometry.
- VII. Measurement.
- VIII. Informal geometry I (Angle relationships).
- IX. Informal geometry II.
- X. Approximation.
- XI. Mathematics in science.
- XII. Statistics.
- XIII. Chance.
- XIV. Finite mathematical systems.

These units are presently being revised on the basis of the comments and suggestions given by the teachers of the various units. Most reports reflect the feeling that the success of these experimental units is a reality. It is recognized that there are various possible ways to organize the proposed subject matter. Investigations mentioned in chapter II of this report reveal that the writers of textbooks are developing the content in accordance with their own philosophy of secondary education. While there may be other adequate outlines, the topics covered in this outline, when well taught by competent teachers, may well serve to give the 7th and 8th grade student an adequate foundation for the modern mathematics program of the secondary school.

#### Mathematics for grade 9:

The new emphasis in the study of algebra is upon the understanding of the fundamental ideas and concepts of the subject, such as the nature

of number systems and the basic laws of operations. The goals of instruction in algebra should not be thought of exclusively as the development of manipulative skills. Rather, it should be oriented toward the development and understanding of properties of a number field.<sup>2</sup> At this point it is clearly emphasized that it is not an alternative of either skill or understanding, for both skills and concepts are essential. The skills must be based on understanding and not merely on rote memorization. Once meaning has been achieved, then drill should be provided to establish the necessary skills. When algebra is studied or taught with an eye to the fundamental nature of the subject, this more general point of view tends to make the whole subject more understandable.

The units listed for grade 9 (algebra) are:

- I. Algebraic structure and the numbers of arithmetic.
- II. Real number system.
- III. Coordinate and linear equations and inequalities in one unknown.
- IV. Variation.
- V. Functions.
- VI. Linear equations and inequalities in two variables.
- VII. Polynomial expressions.
- VIII. Rational (Fractional) expressions.
- IX. Informal deduction in algebra.
- X. Quadratic equations.

Mathematics for grade 10 (geometry).

Three main objectives underlie the teaching of geometry in the secondary school:

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<sup>2</sup>Program for College Preparatory Mathematics, Report of Commission on Mathematics, College Entrance Examination Board, (New York, 1959), p. 20.

1. Acquisition of information about geometric figures in the plane and in space.
2. Development of and understanding of the deductive method as a way of thinking, and a reasonable skill in applying this method to mathematical situations.
3. Provision of opportunity for original and creative thinking by students.

One of the most important characteristics of both traditional and modern mathematics has been an emphasis on deduction. Deductive reasoning has always been regarded as the mathematical method. Perhaps it was in geometry that the modern point of view was first expressed. In the study of geometry, the student should develop an appreciation of the facts and principles concerning space and a knowledge of the basic formulas.

The topics and sequence are as follows:

- I. Informal geometry.
- II. Deductive reasoning.
- III. Sequence of theorems.
- IV. Coordinate geometry.
- V. Additional theorems.
- VI. Solid geometry.

Mathematics for grade 11.

This section is divided into a one semester course in algebra and a one semester course in trigonometry.

A result of the recent growth of mathematics has been the reorganization, extension and transformation particularly of algebra. The essence of this transformation is essentially that algebra is now thought of as the study of mathematical structure and is concerned with patterns of



thought and insights into meaning. Thus, contemporary mathematics is characterized by: (1) a tremendous development quantitatively, (2) the introduction of new content, (3) the reorganization and extension of older content, and (4) renewed, increased and conscious emphasis on the view that mathematics is concerned with patterns of thought.

The sequence of units is as follows:

- I. Basic concepts and skills.
- II. Linear functions, equations and graphs.
- III. Systems of linear equations in one, two and three unknowns.
- IV. Radicals.
- V. Quadratic functions, equations and graphs.
- VI. Systems of quadratic equations.
- VII. Exponents and logarithms.
- VIII. Series.
- IX. Algebraic structure and number fields.

The basic idea of plane trigonometry is the study of the relationship between the sides of a right triangle, taken two at a time. The special names given to these ratios are sine, cosine, tangent, cotangent, secant, and cosecant. The laws of sines and cosines are used in computing triangles which are not right triangles. The value of these ratios depend only on the value of one of the angles opposite the right angle. It is possible, knowing one of the sides and any two other parts of the triangle, to compute the other angles or the length of the other sides.

Trigonometric functions and vectors have many applications in physics and higher mathematics and are important reasons for the inclusion of trigonometry in the high school curriculum. The ideas of vectors or directed magnitudes is, as A. N. Whitehead says "the root-idea of

physical science."<sup>3</sup>

Trigonometric units of study are as follows:

- I. Plane vectors and scalar quantities.
- II. Coordinate trigonometry and vectors.
- III. Trigonometric formulas.
- IV. Trigonometric functions.
- V. Variation, Periodicity and graphs of trigonometric functions.
- VI. Equations and identities.
- VII. Circular functions of sums and differences.
- VIII. Trigonometric analyses.

Mathematics for grade 12: taken from report by Dr. Zant.

Three possible programs are suggested as follows:

1. Elementary functions, first semester; introductory probability with statistical applications, second semester.
2. Elementary functions, first semester; introduction to modern algebra, second semester.
3. Elementary functions and selected topics: elementary functions enlarged to a full year by additional topics.

These three suggested courses are given in outline form only because of the space involved in the complete forms. The course on elementary functions is merely the minimal proposal by the Commission. In the event that a school should be able to complete this program by the end of the first semester of the 12th grade, the Commission recommends that one of the three second-semester alternatives be taken depending on the need.

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<sup>3</sup>A. N. Whitehead, An Introduction to Mathematics, Galaxy Book 18, (New York, 1958), p. 92.

Elementary functions:

- I. Sets and combinations.
- II. Functions and relations.
- III. Polynomial functions.
- IV. Exponential functions.
- V. Logarithmic functions.
- VI. Circular functions.

The following are the three second-semester alternatives in outline form only. A fuller list of content may be found in the Commission's Report, pages 43-47.

Elementary Probability with Statistical Applications:

- I. The nature of probability and statistics.
- II. Organization and presentation of data.
- III. Summarizing a set of measurements.
- IV. Intuitive approaches to probability.
- V. Formal approach to probability.
- VI. The law of chance for repeated trials.
- VII. Applications of binomial distributions.
- IX. The laws of uncertainty.
- X. Relations between two variables.

Introduction to modern algebra:

- I. Fields.
- II. Ordered fields.
- III. Abelian groups.
- IV. Transformation; composition.
- V. Groups (not necessarily commutative).

Selected Topics (to follow Elementary Functions):

- VI. Additional work on sets, functions and relations.
- VII. Further treatment of mathematical induction.
- VIII. Further treatment of permutations, combinations and the binomial theorem.
- IX. Probability.
- X. Additional work on inequalities and absolute values, solution sets and graphs.
- XI. Graphing of factorable polynomials and rational functions.
- XII. Systems of equations with emphasis on graphical interpretation.
- XIII. Coordinate geometry of three dimensions (planes, lines, spheres).

## CHAPTER V

### THE EMERGING STUDENT

The success of a mathematics program at the secondary school level, or at any other level, does not come from the multiplicity of courses offered; nor does it come when a great deal of money is spent per pupil; it does not come even when it can be said that great numbers of pupils pass through the courses offered; it does not come from the fact that teachers are well-educated and well-trained, alone. If these and similar things do not mean success, upon what does the success of such a program rest? It must rest upon the pupil: what did he get from exposure to the course materials and how much did he retain for future use and activity on the job?

In the Commonwealth of Pennsylvania, educators in general and mathematics teachers in particular have become increasingly dissatisfied with the ineffectiveness of their mathematics program and have voiced their concern about such problems as:

1. Why is there such a large number of failures in mathematics?
2. What is the reason for the lack of practical applications?
3. Why is there a tendency to reduce the mathematical training of the average pupil?
4. Why do pupils tend to dislike mathematics?
5. Why is content so largely concerned with the abstract and the mechanical?
6. What is the need to have a more unified program?
7. How can instructional aids and materials be used more effectively? Where can we get them?
8. Why is there lack of fundamental preparation at all grade levels?

9. Why is there lack of retention of the materials of mathematics?
10. Are there enough provisions for the varied needs of all pupils?
11. Is there too little coordination of methods used in presenting similar topics by successive teachers?
12. Is there insufficient guidance of various types?
13. How can instruction be individualized?
14. How can meaningful teaching be developed?
15. How can different types of outcomes be evaluated?
16. How can the interest and effort of many pupils be aroused?<sup>1</sup>

Many other states have already made such studies also. Their recommendations are in the process of being tested for approval in schools within the states. Texas, Oklahoma, Illinois, Louisiana, Massachusetts, Wisconsin, Maryland, and others are at present vitally concerned about their present curricula and the place of mathematics and the sciences.

Better mathematics is needed today in order to produce better mathematicians. Henry Van Engen says that we need good mathematics in schools because good mathematics is easier to learn than bad mathematics.<sup>2</sup> Pupils themselves deserve the best mathematics teaching possible. The average secondary school pupil has a very inquisitive mind which needs to be challenged by intriguing problem situations and plausible reasoning. Mathematics is precise and to be good it needs to say precisely what is meant so that it is more easily understood. Pupils who have been properly prepared by explanations, rules, examples, etc. should be expected to learn more easily.

A good healthy why-attitude should be developed at an early age in

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<sup>1</sup>Mathematics for Secondary Schools, Bulletin 360, 1952, Department of Public Instruction, Harrisburg, Pa., p. xiii-xiv.

<sup>2</sup>Henry Van Engen, "If It's Good Mathematics, It's Easier to Learn," The Mathematics Teacher, February, 1957, L, p. 174.

pupils of mathematics. Thus, meaning will be sought in units of study. If the mathematics program is to be made more meaningful, it should find out first of all what the pupils themselves need to know. Perhaps, here lies the fundamental reason why so many pupils complete the minimum course requirements in mathematics with the "never again" attitude. It is thought that the laboratory approach to mathematics would help to make concepts clearer and cause principles to be retained longer; in other words, learning would be more effective.

The goals of all mathematical learning should be the understanding of mathematical concepts, the acquisition of mathematical skills, and the development of patterns of mathematical thinking through which creative quantitative thinking can take place. Perhaps it is the orderly and systematic nature of all mathematics that has permitted it too frequently to be taught by drill with emphasis on its manipulative skills. Today the major concern of teachers of mathematics is how to develop meaning and understanding in all phases of mathematics. But the classroom teacher can do little to improve mathematical learning and to teach for precise mathematical meanings and understandings without a knowledge of what learning is and how it takes place.<sup>3</sup>

There can be no special course for the teaching of mathematical symbols and language. These must be given precisely and clearly as the need arises. Thus, the use of variables, formulas, exponents, manipulations will acquire in the mind of the student reason and relationship. Verbosity and ambiguity have no place in mathematical works. Johnny need not hate mathematics but his mind, gifted or otherwise, may be challenged by ideas and principles that may cause him to enroll gladly in the

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<sup>3</sup> \_\_\_\_\_, Improvement of the Teaching of Mathematics, Oklahoma State Department of Education, June, 1957, p. 51.

next course in the mathematics sequence. If sufficient challenge and motivation is there, not only will he enroll in the course next in the curriculum but he will begin to see that mathematics is not a hodge-podge of unrelated ideas and concepts. Rather, to him, it will be a progressive structural course. Every principle clearly understood becomes a tool of great importance in the mass of mathematical data with which mathematicians deal.



## CHAPTER VI

### SUMMARY

In suggesting a program of courses for modern high school mathematics, the point of view taken by the writers will be basically that of the Commission on Mathematics and the School Mathematics Study Group. It will probably coincide with that of most of the other groups and individuals who have worked on this particular problem.

Factors which seem to be prominent in this problem from the research and investigation conducted for this report are mentioned again in the following paragraphs.

Change was inevitable. The general public and professional concern for the effectiveness of the mathematics program is an indication of the need for a thorough revision of the entire mathematics program from a point of view of presenting the units and topics in ways that will be mathematically fruitful and meaningful to the high school student. If the secondary school is to provide the service for which it was created, it must not only be aware of its purpose and the educational needs it is designed to serve, but it must also adjust its program to meet these needs. America, today, faces a crisis. Advances in science, technology and industry are continually creating new demands and emphasizing the fact that development and applications of mathematics is indispensable to human progress. The traditional mathematics program has failed to produce the attitudes, understandings, skills and appreciations in students necessary

to meet our national need for mathematical manpower. This is a challenging responsibility and is ample reason for change.

The revision needed in our mathematics curricula demands cooperation and effective planning as well as expert and professional knowledge on the part of many individuals and groups. Expecially needed is the voice of teachers, faculty groups and administrators.

The major demands of any program revision rest heavily upon the shoulders of the teachers. The mathematics teacher, therefore, has the responsibility of directing the high school program toward its ultimate goal and also of emphasizing the role of modern mathematics in meeting the needs of our increasingly complex society.

Strong foundational preparation, both in concepts and skills for grades 7 and 8 will give adequate foundation upon which to build the additional concepts and understandings necessary for the student who desires to contribute to this all-out effort on the part of scientists and technologists to survive and to create a better world in which to live.

Algebra should be presented from the point of view that will develop concepts and understanding so that it will emphasize the structure of mathematics.

Deductive reasoning should be stressed throughout the study of mathematics.

In all cases, the development of the modern mathematics program should be preceded by the development of proper attitudes, skills and understanding on the part of the teacher so that whatever is taught will be taught well.

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## VITA

Chester Henderson Bullock

Candidate for the Degree of  
Master of Science

Report: A MODERN PROGRAM IN MATHEMATICS FOR THE SECONDARY SCHOOL

Major Field: Natural Science

### Biographical:

Personal data: Born in Saline County, Arkansas, December 4, 1918,  
the son of Claude and Alice Holiman Bullock.

Education: Attended the public schools of Grant and Columbia  
Counties, Arkansas; graduated from Magnolia High School,  
Magnolia, Arkansas, in 1938; received the Bachelor of Science  
Degree from Southern State College, Magnolia, Arkansas, with  
a major in English, in August, 1952; completed requirements  
for the Master of Science Degree in May, 1960, at Oklahoma  
State University, Stillwater, Oklahoma.

Professional experience: Taught in the public schools of Arkansas,  
1943-1948; taught in Buhl High School, Buhl, Idaho, 1948-1954;  
taught in Magnolia High School, Magnolia, Arkansas, 1954-1959.

Professional organizations: Member of the National Education  
Association; Arkansas Education Association; Arkansas  
Mathematics Teachers; Classroom Teachers Organization.

## VITA

Louis Malcolm Collier

Candidate for the Degree of

Master of Science

Report: A MODERN PROGRAM IN MATHEMATICS FOR THE SECONDARY SCHOOL

Major Field: Natural Science

### Biographical:

Personal data: Born in Little Rock, Arkansas, May 19, 1919, the son of Albert and Ludia Collier.

Education: Attended grade school and graduated from Lincoln High School, Ruston, Louisiana, in 1937; received the Bachelor of Science Degree from Grambling College, Grambling, Louisiana, with a major in Science Education, in May, 1954; completed requirements for the Master of Science Degree in May, 1960, at Oklahoma State University, Stillwater, Oklahoma.

Professional experience: Taught junior and senior high school mathematics and science for five years in the public schools of Louisiana.

Professional organizations: Member of the National Education Association; National Science Teachers Association; National Council of Teachers of Mathematics; Louisiana Education Association; Louisiana Institute of Science; Quachita Parish Education Association.